

## Related topics

Angular velocity, characteristic frequency, resonance frequency, torsional pendulum, torsional oscillation, restoring torque, damped/undamped free oscillation, forced oscillation, ratio of damping/decrement, constant damping, logarithmic decrement, aperiodic case, and creeping

## Principle

If an oscillating system is allowed to oscillate freely, we can observe that the decrement of successive maximum amplitudes strongly depends on the damping value. If the oscillating system is caused to oscillate by an external torsional oscillation, we can observe that the amplitude in a stationary state is a function of the frequency and amplitude of the external periodic torsional oscillation and of the damping value. The aim of this experiment is to determine the characteristic frequency of the free oscillation as well as the resonance curve of a forced oscillation.

## Equipment

1	torsional pendulum (after R.W. Pohl)	11214-00
1	variable transformer, 25 V AC/20 V DC, 12 A	13531-93
1	bridge rectifier, 30 V AC/1 A DC	06031-10
1	digital multimeter	07129-00
2	connecting cords, l = 250 mm, yellow	07360-02
2	connecting cords, l = 750 mm, red	07362-01
3	connecting cords, l = 750 mm, blue	07362-04
1	"measure Dynamics" software	14440-61

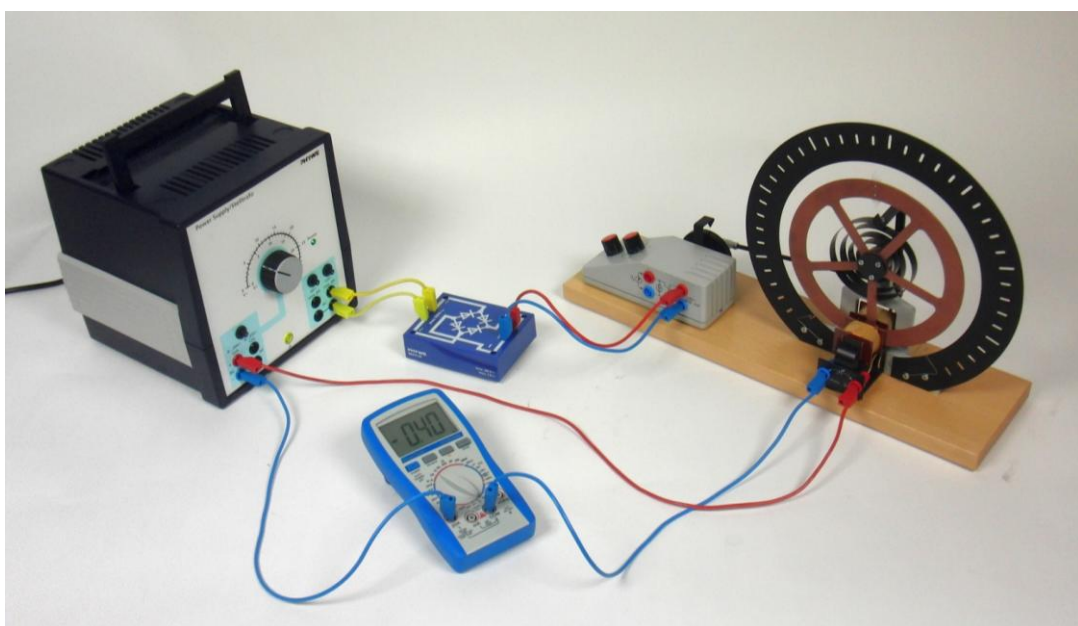


Figure 1: Experiment set-up

### Additional material:

Coloured, circular disc, video camera, tripod, computer

### Tasks

#### A. Free oscillation

1. Determination of the period of oscillation and characteristic frequency for the undamped case.
2. Determination of the period of oscillation and corresponding characteristic frequencies for various damping values. Calculation of the corresponding ratios of damping, damping constant, and logarithmic decrement.
3. Realisation of the aperiodic case and creeping case.

#### B. Forced oscillation

1. Determination of the resonance curve and graphical representation of the resonance curve by way of the damping values of A.
2. Observation of the phase difference between the torsional pendulum and the exciting, external rotation for a small damping value with different excitation frequencies.

### Set-up

Set the experiment up as shown in Figures 1 and 2. Connect the DC output of the variable transformer to the eddy current brake. The excitation motor also requires DC voltage, which is why a bridge rectifier must be connected between the AC output (12 V) of the variable transformer and the excitation motor (DC motor, see Figure 3). Connect the DC current  $I_B$ , which supplies the eddy current brake, via the adjusting knob of the variable transformer and check it by way of an ammeter.

In terms of the video that will be recorded, the following must be taken into consideration concerning the setting and positioning of the camera:

- Set the number of frames per second to approximately 30 fps.
- Select a light-coloured, homogeneous background.
- Provide additional lighting for the experiment.
- The experiment set-up, i.e. Pohl's pendulum, should be in the centre of the video. To ensure this, position the video camera on a tripod centrally in front of the experiment set-up. For the experiment concerning the forced oscillations, however, it is important that the excitation motor is also filmed. However, the pendulum must remain in the centre of the video.
- The experiment set-up should fill the video image as completely as possible.

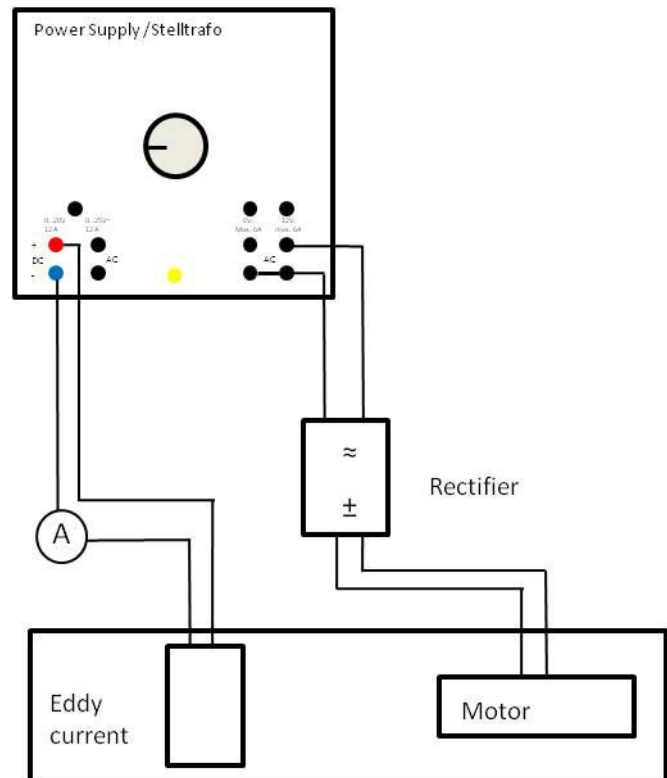


Figure 2: Power connection for the experiment

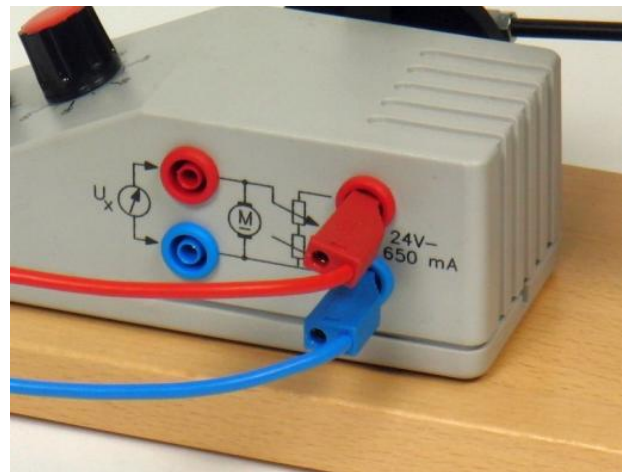


Figure 3: Connection of the excitation motor (DC motor) to the variable transformer

- The optical axis of the camera must be parallel to the experiment set-up.
- For the video analysis, we recommend gluing a circular, coloured sheet of paper onto the rotating needle of the pendulum (see Figure 4).
- Measure the width of the pendulum for scaling.

Then, the video recording process and the experiment can be started.



Figure 4: Pendulum pointer in the zero position

## Procedure

### A. Free oscillation

#### 1. Determination of the period of oscillation and characteristic frequency for the undamped case.

In order to determine the characteristic frequency  $\omega_0$  of the torsional pendulum without damping ( $I_B = 0$ ),

- start the video recording process,
- deflect the pendulum completely in one direction,
- measure the time that is needed for several oscillations, and
- stop the video recording process.

#### 2. Determination of the period of oscillation and corresponding characteristic frequencies for various damping values.

Prior to starting the experiment, it must be ensured that the pointer of the pendulum at rest coincides with the zero position of the scale (see Figure 4). This can be achieved by manually turning the eccentric disc of the motor.

In the same manner as for A1, the characteristic frequencies for damped oscillations must be determined. These frequencies are caused by the following currents flowing through the eddy current brake:

$$I_B \sim 0.45 \text{ A}$$

$$I_B \sim 0.80 \text{ A}$$

The damping values can then be determined based on the recorded video.

#### 3. Realisation of the aperiodic case and creeping case.

Adjust these currents in order to realise the aperiodic case ( $I_B \sim 2.0 \text{ A}$ ) and the creeping case ( $I_B \sim 2.3 \text{ A}$ ). The rest of the experiment procedure is the same as for A1 and A2.

Caution: Current intensities above 2.0 A must not be used for more than a few minutes for the eddy current brake!

### B. Forced oscillation

In order to excite the torsional pendulum, the connecting rod of the motor must be fastened to the upper third of the source of excitation. The excitation frequency  $\omega_a$  can be determined by determining the time that is required for several rotations of the motor in the video.

#### 1. Determination of the resonance curves and graphical representation of the resonance curves by way of the damping values of A.

The (continuous) video recording process begins with low excitation frequencies  $\omega_a$ . Then, increase  $\omega_a$  first with the "coarse" motor potentiometer. Close to the maximum of  $\omega_a$ , vary the excitation frequency in small steps with the "fine" motor potentiometer (see Figure 5). In each case, ensure that the pendulum amplitude is stable prior to varying the excitation frequency. In the case of a pendulum movement without damping or with only very small damping values, the excitation frequency  $\omega_a$  must be chosen so that the pendulum does not exceed its maximum deflection.

#### 2. Observation of the phase difference between the torsional pendulum and the exciting, external rotation for a small damping value with different excitation frequencies.

Choose a small damping value and excite the pendulum with a frequency  $\omega_a$  far below and then far above the resonance frequency. Record these processes by way of the video camera.

## A. Undamped and damped free oscillation

### Theory

In the case of a free, damped torsional oscillation, the torques  $M_1$  (spiral spring) and  $M_2$  (eddy current brake) act upon the pendulum. We have

$$M_1 = -D^0\phi \text{ and } M_2 = -C\dot{\phi}$$

$\phi$  = angle of rotation

$\dot{\phi}$  = rotational velocity

$D^0$  = rotation per unit angle

$C$  = proportionality factor, depending on the currents that flow through the eddy current brake

The resulting torque

$$M_1 = -D^0\phi - C\dot{\phi}$$

leads us the following equation of motion:

$$I\ddot{\phi} + C\dot{\phi} + D^0\phi = 0 \quad (1)$$

$I$  = moment of inertia of the pendulum

$\ddot{\phi}$  = angular acceleration



Figure 5: Control knobs for adjusting the excitation frequency. Upper knob: "coarse"; lower knob: "fine".

Dividing equation (1) by  $I$  and using the abbreviations

$$\delta = \frac{C}{2I} \text{ and } \omega_0^2 = \frac{D^0}{I}$$

results in

$$\ddot{\phi} + 2\delta\dot{\phi} + \omega_0^2\phi = 0 \quad (2)$$

$\delta$  is known as the "damping constant" and

$$\omega_0 = \sqrt{\frac{D^0}{I}}$$

is the characteristic frequency of the undamped system. The solution of the differential equation (2) is

$$\phi(t) = \phi_0 e^{-\delta t} \cos \omega t \quad (3)$$

with

$$\omega = \sqrt{\omega_0^2 - \delta^2} \quad (4)$$

## Evaluation

Transfer the various videos that have been recorded to the computer. Then, start "measure Dynamics" and open the video under "File" – "Open video ...". Mark the start of the experiment ("Start selection" and "Time zero") and the end of the experiment ("End selection") in the video for further analysis via the menu line above the video. The experiment begins when the torsional pendulum is released and it ends when the video ends. Mark the width of Pohl's pendulum with the scale that appears in the video by way of "Video analysis" – "Scaling ..." – "Calibration" and enter the measured length (in this case 0.30 m) into the input window. In addition, enter the frame rate that has been set for the recording process under "Change frame rate" and position the origin of the system of coordinates in the centre of rotation of the pendulum under "Origin and direction".

Then, the actual motion analysis can be started under "Video analysis" – "Automatic analysis" or "Manual analysis". For the automatic analysis, we recommend selecting "Motion and colour analysis" on the "Analysis" tab. Under "Options", the automatic analysis can be optimised, if necessary, e.g. by changing the sensitivity or by limiting the detection radius. Then, look for a film position in the video where the object that is to be analysed is perfectly visible. Click the object. If the system recognises the object, a green rectangle appears and the analysis can be started by clicking "Start".

If the automatic analysis does not lead to any satisfying results, the series of measurements can be corrected under "Manual analysis" by manually marking the object that is to be analysed.

*Task A1: Determination of the period of oscillation and characteristic frequency for the undamped case.*

In order to determine the average period of oscillation  $\bar{T}_0$  and the corresponding characteristic frequency  $\bar{\omega}_0$  of the free and undamped, oscillating torsional pendulum, we have to determine the time that the pendulum needs for several oscillations (e.g. 10 oscillations). This leads to:

$$\bar{T}_0 = 1.962 \text{ s}$$

and

$$\bar{\omega}_0 = 3.20 \cdot \frac{1}{\text{s}}$$

*Task A2: Determination of the period of oscillation and corresponding characteristic frequencies for various damping values. Calculation of the corresponding ratios of damping, damping constant, and logarithmic decrement.*

The average period of oscillation  $\bar{T}_0$  and the average frequency  $\bar{\omega}_0$  are determined in the same manner as for task 1. For  $I_{B,1} = 0.45 \text{ A}$ , this leads to

$$\bar{T}_{0,1} = 1.946 \text{ s}$$

$$\bar{\omega}_{0,1} = 3.23 \cdot 1/\text{s}$$

For  $I_{B,2} = 0.80 \text{ A}$ , this leads to

$$\bar{T}_{0,2} = 1.935 \text{ s}$$

$$\bar{\omega}_{0,2} = 3.25 \cdot 1/\text{s}$$

Then, perform a graphical representation of the maximum amplitudes on one side of the pendulum, i.e. always on the right-hand side or always on the left-hand side of the pendulum, as a function of time. Add a new column to the worksheet by clicking "New column" in the table menu line. Enter "Phi" (unit: "°"; formula: "arctan2(x;y)\*360/(2\*π)") into this new column. Then, create another column (name: "Phi\_max"; unit: "°"). Find the various maxima of "Phi" in the table and enter the values of "Phi" of these maxima into the column "Phi\_max". In order to display the maxima of "Phi" as a function of time, select "Display" and "Diagram", click "Options", delete all of the already existing graphs, and select the graphs t (horizontal axis) – "Phi\_max" (vertical axis). For  $I_{B,1} = 0.45 \text{ A}$ , this leads to:



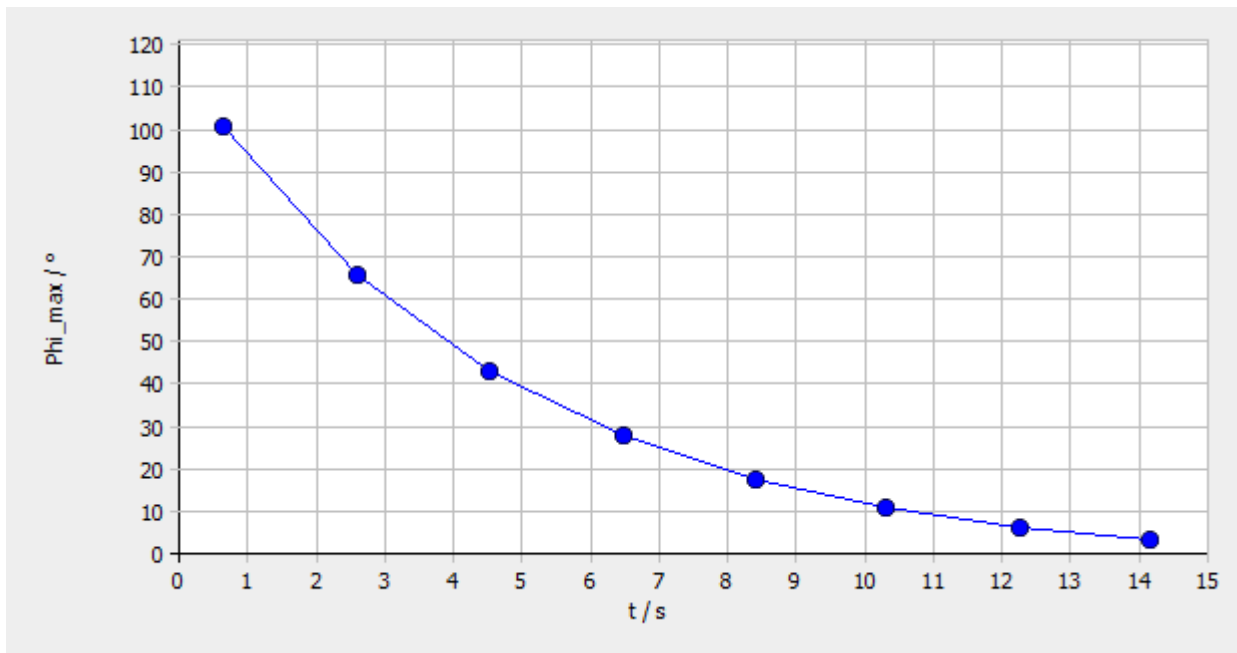


Figure 6: Representation of the maximum deflections of the pendulum for  $I_{B,1} = 0.45 \text{ A}$  on one side as a function of time  $t$

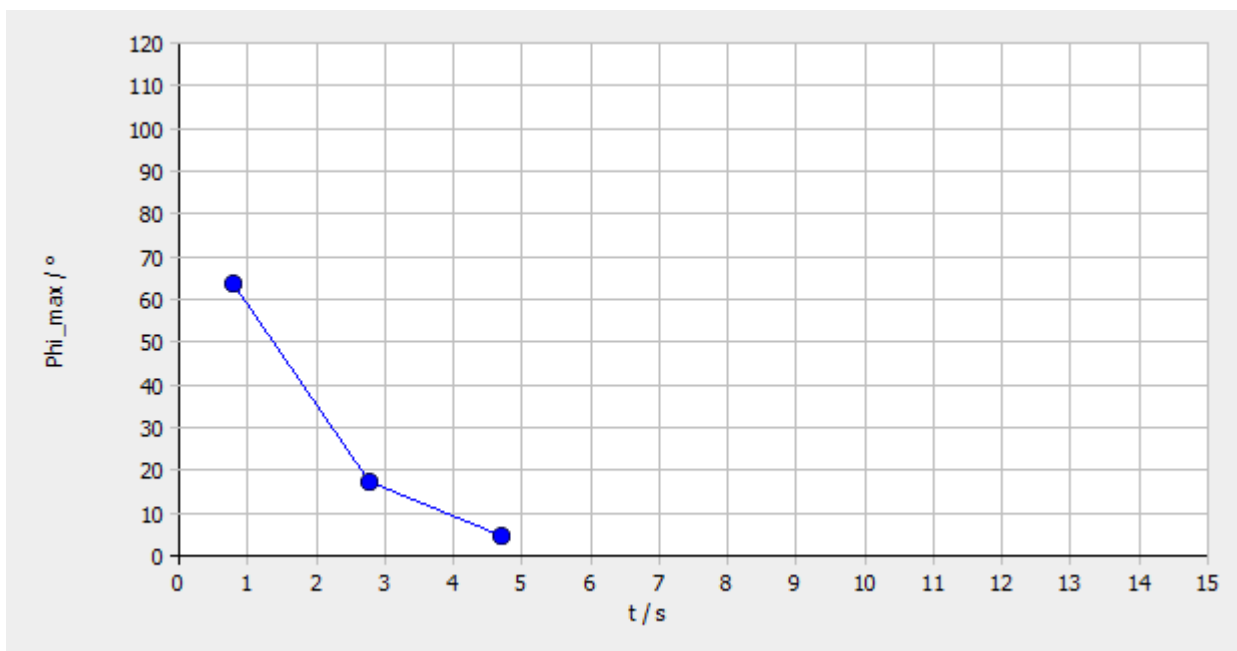


Figure 7: Representation of the maximum deflections of the pendulum for  $I_{B,2} = 0.80 \text{ A}$  on one side as a function of time  $t$

For  $I_{B,2} = 0.80 \text{ A}$ , this leads to:

We already know from the theory – and Figures 6 and 7 confirm this – that a higher current  $I_B$  causes the torsional pendulum to be decelerated more quickly.

The resulting relationship between damping, damping constant  $K$ , and logarithmic decrement  $\Lambda$  is calculated as follows:

Equation (3) shows that the amplitude  $\phi(t)$  of the damped oscillation decreases to the  $e^{\text{th}}$  part of the initial amplitude  $\phi_0$  after the time  $t = 1/\delta$ . Equation (3) also shows that the ratio of two successive amplitudes is constant.

$$\frac{\phi_n}{\phi_{n+1}} = K = e^{\delta T} \quad (5)$$

$K$  is known as the damping constant or "damping ratio",  $T$  as the period of oscillation, and

$$\Lambda = \ln K = \delta T = \ln \frac{\phi_n}{\phi_{n+1}} \quad (6)$$

as the "logarithmic decrement". This leads to:

Table 1:

$I/A$	$\frac{1}{\delta}/s$	$\delta/s^{-1}$	$\omega = \sqrt{\omega_0^2 - \delta^2}/s$	$K = \frac{\phi_n}{\phi_{n+1}}$	$\Lambda$
0.45	4.6	0.22	3.21	1.54	0.43
0.80	1.5	0.67	3.27	3.72	1.31

The difference of the values for the frequency  $\omega$  is due to the fact that  $\delta$  can be determined only in a rather imprecise manner. Still, the calculated frequencies are nearly consistent with the measured frequencies.

*Task A3: Realisation of the aperiodic case and creeping case.*

Add a new column with the name "Phi" to the table in the same manner as for task A2. Select "Display"

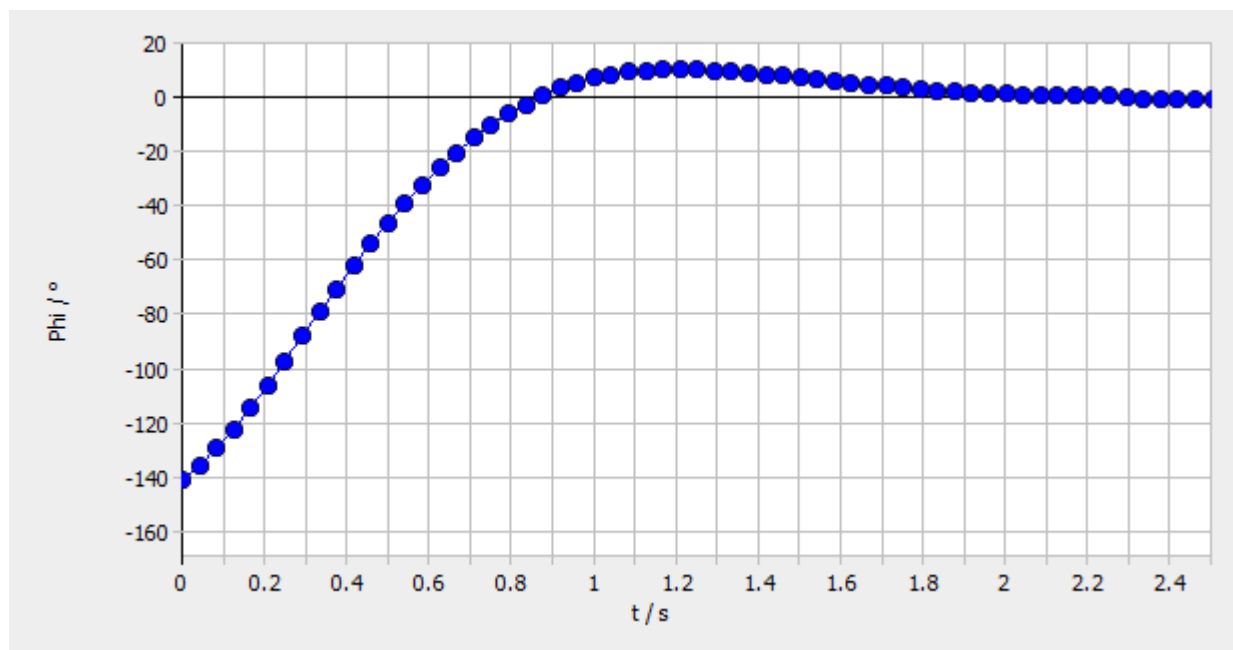


Figure 8: Representation of the deflections of the pendulum for  $I_B = 2.00$  A as a function of time  $t$



and "Diagram", click "Options", delete all of the already existing graphs, and select the graphs  $t$  (horizontal axis) – " $\Phi$ " (vertical axis). This leads to:

According to the theory, equation (4) has a real solution only if  $\omega_0^2 \geq \delta^2$ . For  $\omega_0^2 = \delta^2$ , the pendulum returns to its initial position very quickly without oscillation (aperiodic case, see Figure 8). For  $\omega_0^2 < \delta^2$ , the

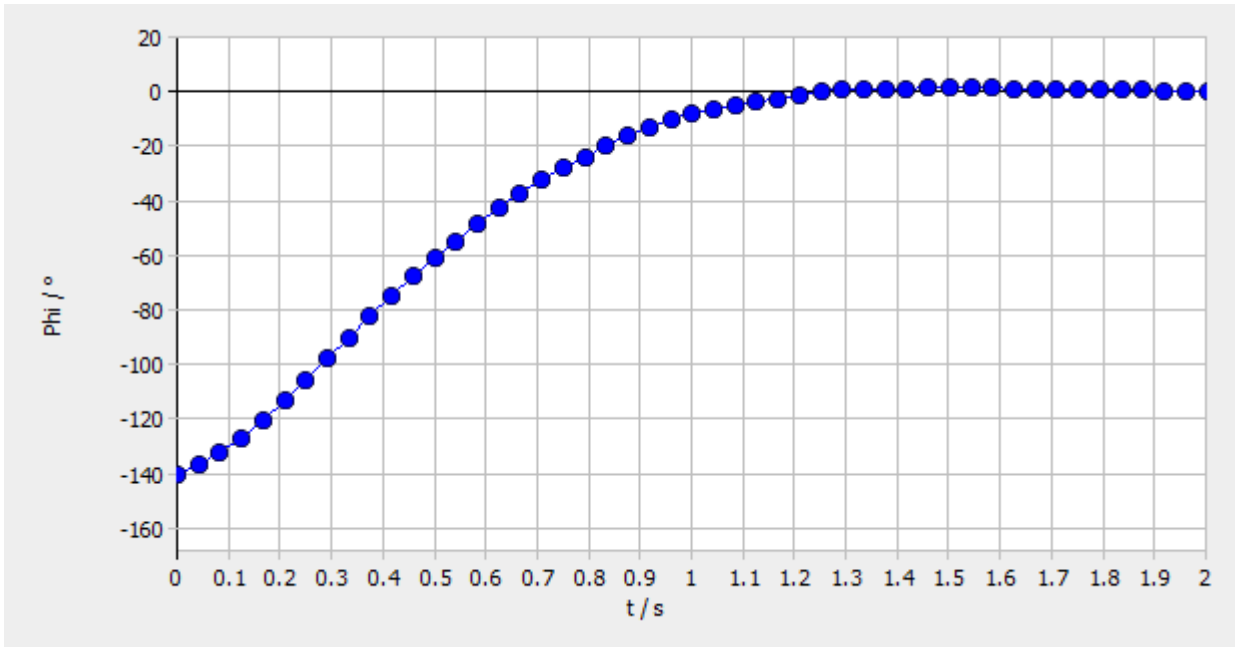


Figure 9: Representation of the deflections of the pendulum for  $I_B = 2.30 \text{ A}$  on one side as a function of time  $t$  pendulum returns to its initial position in an asymptotic manner (creeping case, see Figure 9).

## B. Forced oscillation

### Theory

If the pendulum is subject to a periodic torque  $M_a = M_0 \cos \omega_a t$ , equation (2) changes to

$$\ddot{\phi} + 2\delta\dot{\phi} + \omega_0^2\phi = F_0 \cos \omega_a t \quad (7)$$

where  $F_0 = \frac{M_0}{l}$ . In the stationary state, the solution of the differential equation is

$$\phi(t) = \phi_a \cos(\omega_a t - \alpha) \quad (8)$$

with

$$\phi_a = \frac{\phi_0}{\sqrt{\left\{1 - \left[\frac{\omega_a}{\omega_0}\right]^2\right\}^2 + \left[2\frac{\delta}{\omega_0}\frac{\omega_a}{\omega_0}\right]^2}} \quad (9)$$

and  $\phi_a = \frac{F_0}{\omega_0^2}$

Furthermore,

$$\tan \alpha = \frac{2\delta\omega_a}{\omega_0^2 - \omega_a^2}$$

and also

$$\alpha = \arctan \frac{2\delta\omega_a}{\omega_0^2 - \omega_a^2}. \quad (10)$$

### Evaluation

*Task B1: Determination of the resonance curve and graphical representation of the resonance curve by way of the damping values of A.*

For this task, we recommend performing the analysis of the rotational motion manually, since only a few moments of the video area actually relevant for the evaluation. Add new columns for "Phi" (see task A2), for the period of oscillation "T" (unit: "s"), and for the excitation frequency "wa" (unit: "1/s", formula: "2\*π/T") to the worksheet. It may be necessary to add a minus sign to the formula when creating the "Phi" column so that the values become positive. Then, determine the excitation frequency (in the same manner as for task A1, but in this case observe the eccentric disc and not the pendulum). Wait until the torsional pendulum has developed a constant amplitude and mark the pendulum at the maximum in the manual analysis. Enter the period of oscillation, which has already been determined, into the same line under "T". Do the same for the other excitation frequencies.

In order to display the maximum deflection "Phi" as a function of the excitation frequency, select "Display" and "Diagram", click "Options", delete all of the already existing graphs, and select the graphs (horizontal axis) – "Phi" (vertical axis). This leads to:

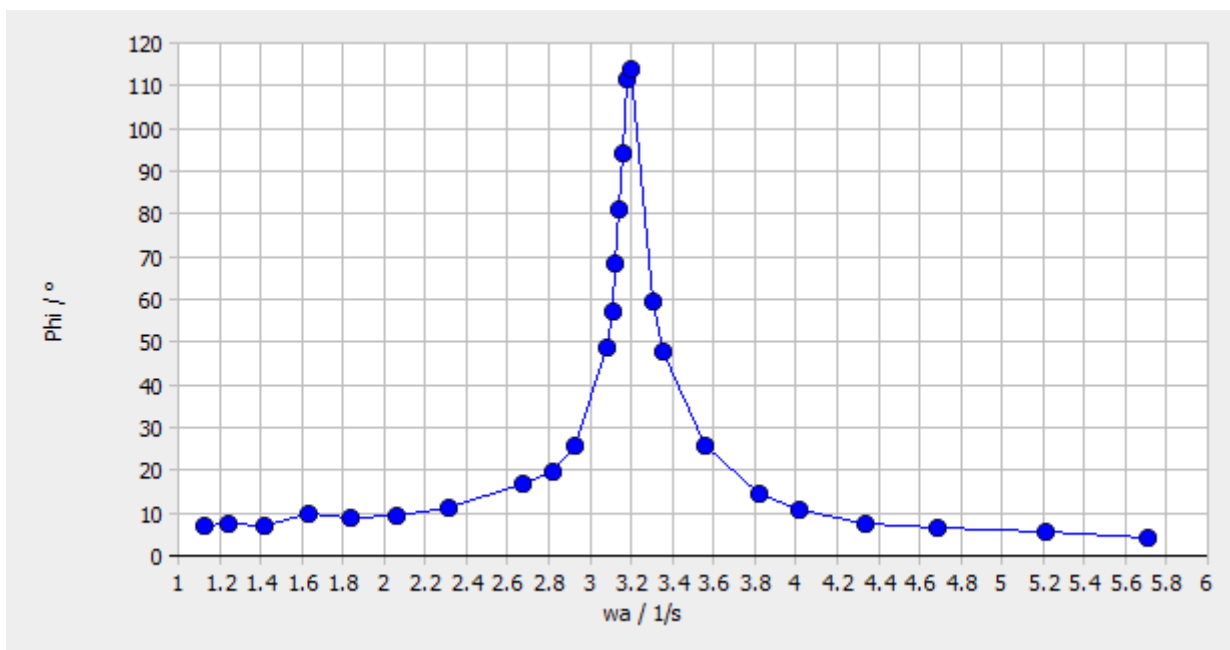


Figure 10: Resonance curve for  $I_B = 0.23 \text{ A}$

Figure 10 shows that the amplitude is very small in the case of low excitation frequencies. When the excitation frequency is slowly increased, a maximum can be observed. Increasing the excitation frequency further leads to smaller amplitudes until the amplitude drops nearly to 0. This is the characteristic shape

of a resonance curve. The relatively sharp maximum of the amplitude occurs at the so-called resonance frequency. Here, the resonance frequency is  $\omega = 3.20 \text{ 1/s}$ . In addition, it becomes clear that the resonance frequency matches the characteristic frequency of the torsional pendulum, which has been determined in task A1. This gives rise to the suspicion that  $\phi \rightarrow \phi_{\max}$  für  $\omega_a \cong \omega_0$ . If one considers equation (9), this suspicion is confirmed by the theory. For higher damping values, the curve is above the represented curve, and it is below the represented curve for smaller damping values.

*Task B2: Observation of the phase difference between the torsional pendulum and the exciting, external rotation for a small damping value with different excitation frequencies.*

For this task, the video of task B1 is required. If we observe the torsional pendulum for a lower excitation frequency  $\omega_a$ , it becomes clear that there is no phase difference between the torsional pendulum and exciting pendulum, i.e. the pendulum and the exciting torque are "in phase". If the excitation frequency  $\omega_a$  is much higher than the resonance frequency  $\omega_0$ , the pendulum and the exciting torque show a phase difference  $\pi$ , i.e. they are in "antiphase" with each other. If the resonance frequency is applied to the torsional pendulum, the phase difference is  $\pi/2$ . The smaller the damping value is, the quicker is the transition from "in phase" oscillation to "antiphase" oscillation.

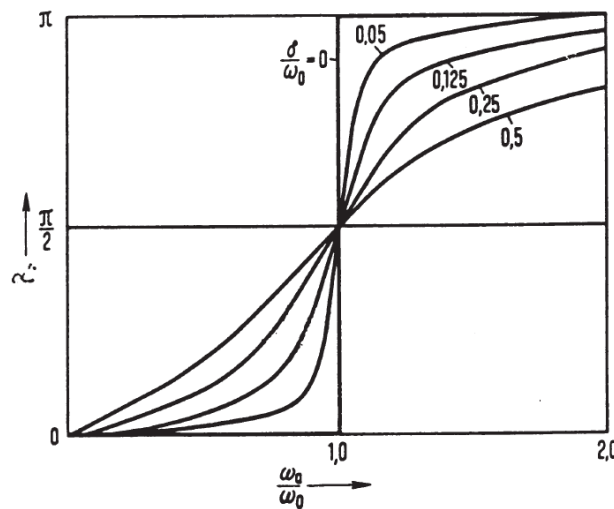


Figure 11: Phase difference in the case of a forced oscillation for different damping values

Space for notes: